Entanglement and the Bell Test of Local Realism

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Experiment carried out at Haverford College as part of PHYS H326
in Oct - Dec 2022.
Submitted: Dec 16, 2022.

Abstract

The discrepancy between the theories of local realism and quantum mechanics spurred huge debates over several decades since 1935 [3]. In 1964, however, John Bell proposed an experimental scheme to directly test the theory of local realism by introducing the so-called "Bell inequalities," which hold in only *one* of the two theories. Since then, it has been experimentally demonstrated numerous times that local realism does not hold. In this experiment, we reproduced one of such experimental efforts to demonstrate that entangled states can violate local realism. We used one version of Bell's inequalities given by Clauser, Horne, Shimony, and Holt (CHSH), which states that the CHSH parameter, S, they defined must be less than 2 for local realism to hold. We prepared pairs of photons in an entangled state. Our measured S was 2.067 ± 0.053, strictly above 2, which indicated that we had achieved successful entanglement. We repeated the same measurement with pairs of photons in an unentangled, mixed state. This gave $S = 1.250 \pm 0.0041$, which satisfied Bell's inequality and thus showed that the photons in the beam were no longer entangled. Our results are in agreement with previous experimental efforts and re-confirm the theory of quantum mechanics and thus the fundamentally probabilistic character of nature.

1 Introduction

The theory of local realism contends two things: First, the properties of an object, such as the wavelength and polarization of a photon, are definite, that is, they are not probabilistic. Second, the information about a system stays "local" and cannot travel faster than the speed of light. However, the theory of quantum mechanics states totally different things: the state of a particle is in *superposition* of multiple different deterministic properties, such as different polarizations of a photon, and gets mapped onto one of those deterministic properties probabilistically upon measurements. Note that this is fundamentally different from a statistical ensemble of photons all having different deterministic properties. Moreover, there is the concept of "entanglement," in which two different systems or particles contain information about one another and can exchange that information *instantaneously*, faster than the speed of light. Although the theory of quantum mechanics had been confirmed by a range of experiments, there was no suitable way to disprove the theory of local realism. For this reason, these two contrasting theories caused huge debates since 1935 [3].

The debates continued until 1964 when John Bell devised an experiment to test the two different philosophical possibilities using the so-called "Bell inequalities," the conditions that hold by only one of the two theories. And using one of the versions of the Bell inequalities, Clauser, Horne, Shimony, and Holt (CHSH) proposed an experimentin 1969 to test the theory of local realism with photons. Their method uses the correlation between the polarizations of a pair of photons [2]. They introduced a variable E, which we call the "correlation parameter," that represents the degree of the polarization correlation between the two photons. If the photons are perfectly correlated then $E = 1$, while we get $E = 0$ if they are perfectly uncorrelated. The correlation parameter E depends on the polarization angles of the photons, $E(\theta_1, \theta_2)$, where θ_1, θ_2 indicate the polarization angles of the first photon and the second photon, respectively.

CHSH then defined a variable S, which we call in this report the "CHSH parameter," that depends on four polarization angles, given by

$$S = E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2'), \tag{1}$$

where θ_1, θ_1' are the two polarization angles of the first photon, and θ_2, θ_2' are the two polarization angles of the second photon.

In the theory of local realism, this CHSH parameter must satisfy

$$|S| \leq 2. \tag{2}$$

This famous result is called the "CHSH Inequality." Entangled photons measured at certain polarization angles violate this inequality, and one of such entangled states is given by the Bell state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|HH\rangle + |VV\rangle \right). \tag{3}$$

Here H and V indicate the horizontal and vertical polarization states of the photon, respectively, and the Dirac notation $|HH\rangle$ represents the state where both the first and the second photons are in the horizontal polarization state. This is an entangled state because it is "non-separable," meaning that the state cannot be factorized into a product of the state of each photon.

The entangled state (Equation 3) can be rewritten in the diagonal D and antidiagonal A polarization basis as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|DD\rangle + |AA\rangle \right), \tag{4}$$

where we have used the relations

$$|D\rangle = \frac{1}{\sqrt{2}} \left(|H\rangle + |V\rangle \right), \tag{5}$$

$$|A\rangle = \frac{1}{\sqrt{2}} \left(-|H\rangle + |V\rangle \right). \tag{6}$$

Using Dirac notations, we can easily see that the probability of jointly detecting the first photon in the diagonal state and the second in the antidiagonal state, or vice versa, is zero:

$$P_{\mathrm{DA}} = |\langle\Psi|DA\rangle|^2 = 0 \tag{7}$$

$$P_{\mathrm{AD}} = |\langle\Psi|AD\rangle|^2 = 0. \tag{8}$$

We used this fact in our experiment to demonstrate the entanglement of two photons.

To test the CHSH inequality experimentally, we need to prepare pairs of entangled photons. Some types of crystals are capable of producing entangled pairs of photons via "spontaenous parametric down-conversion" [1]. One type of such crystals is the Beta Barium Borate (BBO) crystal. In a simple picture, spontaneous parametric down-conversion converts an incoming parent photon of short wavelength to a pair of correlated photons of twice the original wavelength, which have the same energy, magnitude of momentum, and polarization. Figure 1 shows a schematic diagram of the down-conversion process inside a BBO crystal. The pair of down-converted photons satisfy

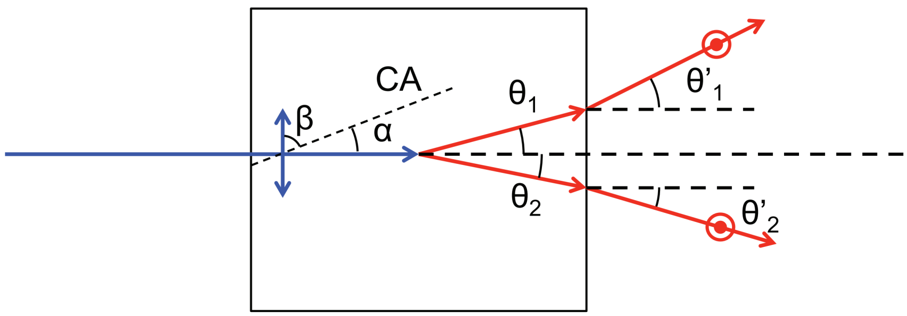

Figure 1: Spontaneous parametric down-conversion in a BBO crystal. CA indicates the crystal axis, which forms an angle β from the polarization (the direction of the electric field) of the parent photon and forms an angle α with the propagation direction. The propagation axes of the down-converted photons make angles θ_1, θ_2 from the parent photon's propagation direction within the crystal, and angles θ'_1, θ'_2 outside the crystal.

the following properties: First, the sum of their energies E_1, E_2 equals the energy E_p of the pump photon:

$$E_p = E_1 + E_2, \tag{9}$$

or equivalently,

$$1/\lambda_p = 1/\lambda_1 + 1/\lambda_2. \tag{10}$$

For down-conversion to occur, momentum must be conserved. Using the conservation of momentum and assuming $\lambda_1 = \lambda_2$, we have $\theta_1 = \theta_2$ and thus obtain the following simple relation

$$n_p = n_1 \cos\theta_1, \tag{11}$$

where n_p and n_1 are the indices of refraction for the pump photon and the down-converted photon, respectively. Using this relation, we can predict the exact angles at which the down-converted photons will be created, if we know the wavelength of the pump beam and the orientation of the crystal axis. Note that the real picture is a bit more complicated: extending the same analysis to 3D, we can see that the down-converted photons, in fact, form a *cone* about the pump-beam axis.

We carried out a series of experiments to produce an entangled state that violates the Bell's inequality (Equation 3). We first created down-converted photons from a BBO crystal, prepared pairs of photons in the entangled state, and took 16 independent measurements of photon counts to demonstrate that the entangled state violates the local realism. The paper is organized as follows: Section 2 describes the procedure for our experiment. Section 3 shows the results of our experiment, including the photon counts for the 16 measurements. We also show that the entangled state we prepared violates the Bell's inequality. We conclude in Section 4.

2 Procedure

We broke down our experiment into three different parts. The first part involved creating down-converted photons by passing a blue laser (405 nm) through a BBO crystal. In the second part, we added in a few more optical elements to confirm that we have an ideal single photon source. Finally, we created entangled photons using a similar setup with a different, thinner two-layered

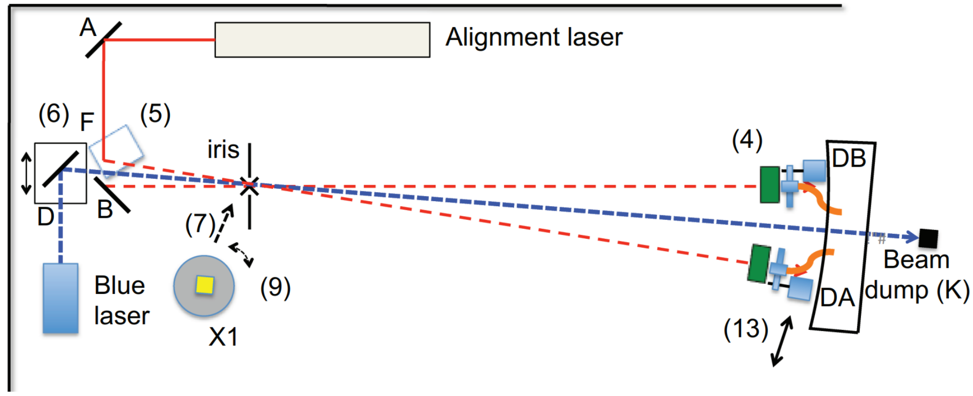

Figure 2: Setup for creating down-converted photons

BBO crystal and performed a series of measurements to directly test the CHSH inequality. The procedure described in this section is based on Chapter 12 of [3].

2.1 Creating Down-Converted Photons

To create down-converted photons, we set up the optic table as shown in Figure 2. We started with an alignment laser (model: Lumentum HeNe HNL008L) and placed two collimators (A and B) exactly 3 degrees apart from each another on the curved plate that had 1 m radius of curvature. Then, we aligned the alignment laser such that when we flip the mirror mount, the beam passes from the center of the first collimator to the second. Finally, the BBO crystal was placed exactly 1 m away from the collimators and at the center of the circle defined by the curved plate. This made it so that the alignment laser passed through the crystal no matter which collimator it was pointed at.

Then, we aligned the collimators by using two techniques: back reflection and back projection. Back reflection involved pointing the alignment laser at the center of the collimator, and making sure its reflection in the collimator's lens passed back though the crystal and into the beam of the laser. Back projection involved attaching a handheld laser source (635 nm; Model: Thorlabs HLS635) to the face of the collimator and passing this laser back into the alignment laser source. Back reflection was also used to align the crystal properly.

Finally, for the pump beam, we used a regular blue (405 nm) laser pointer, which was powered by a DC power supply (model: RSR HY3003). The model number for the blue laser pointer was not available. The images of our pump beam setup are shown in Figure 3. We used a convex lens (f = 1000 mm) to shrink the diameter of the pump beam going through the BBO crystal. This increased the intensity of the pump beam and thus helped getting more down-converted photons. Since the blue laser pointer was unpolarized, we also used a linear polarizer to produce a horizontally polarized light source. The linear polarizer was placed right before the BBO crystal in order to avoid producing any unwanted elliptical polarizations through reflections off the mirrors. Then, the pump beam was passed though the crystal, which caused some of the photons (1 in 10^8 or 10^{10}) to be down-converted. These down-converted photons were detected by the collimators.

Narrow bandpass filters (model: Thorlabs FGL780) were placed in front of the collimators to only detect photons close to the wavelength of down-converted photons (810 nm). While making sure that there was no external light source that could affect the counts, the photon counts were measured with photon detectors (model: Excelitas SPCM-EDU CD3375), and recorded on Lab-

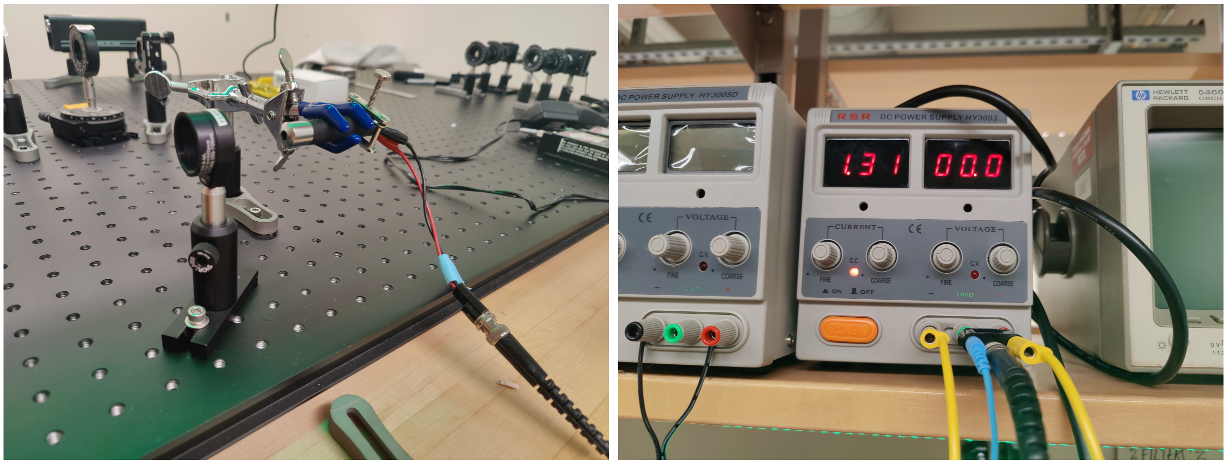

Figure 3: The blue pump beam laser and its power supply.

VIEW. The LabVIEW program we used was provided by [3], and its block diagram and front panel are shown in Figures 4 and 5, respectively. Then, their coincidences were noted. The collimators and the BBO crystal were carefully adjusted vertically and horizontally such that we detect the highest number of photons in each direction.

The accidental coincidences were then calculated by

$$N_{\text{acc}} = \frac{N_1 N_2 \Delta T}{t_d}, \tag{12}$$

where N_1, N_2 are the single counts at the two collimators, ΔT the maximum time delay between pulses that is considered a coincidence, and t_d the total measurement window. For our measurements, we had a fixed $\Delta T = 7$ ns, but we varied t_d. We subtracted these accidental counts from our measured coincidences. Assuming the counts follow a Poissonian distribution, the error of all the counts N were calculated by $\sigma_N = \sqrt{N}$.

2.2 Checking the Existence of Photons

Following from the previous experiment, a third collimator (C) was added, as shown in Figure 6. This new collimator was placed perpendicular off to the side of the first collimator, and a non-polarizing beam splitter cube was used to reflect half the alignment laser light to the center of the third collimator. The alignment of the new collimator was calibrated using back projection and back reflection, following the same methods described in the previous section, and was fine-tuned while we monitored the photon counts on LabVIEW to make sure we had the highest possible photon counts.

Photon counts were measured for detectors A, B, and C, and the coincidences were noted for AB, AC and ABC using LabVIEW. Data was collected for 2 minutes per trial, and we ran 10 trials in total. Then we averaged over the 10 trials and used those average values to calculate the degree of second order coherence.

The degree of second order coherence and its error was calculated by

$$g_2(0) = \frac{N_{\text{ABC}} N_{\text{A}}}{N_{\text{AB}} N_{\text{AC}}} \tag{13}$$

$$\Delta g_2(0) = g_2(0) \left(\frac{1}{N_{\text{ABC}}} + \frac{1}{N_{\text{AB}}} + \frac{1}{N_{\text{AC}}} + \frac{1}{N_{\text{A}}} \right)^{1/2}. \tag{14}$$

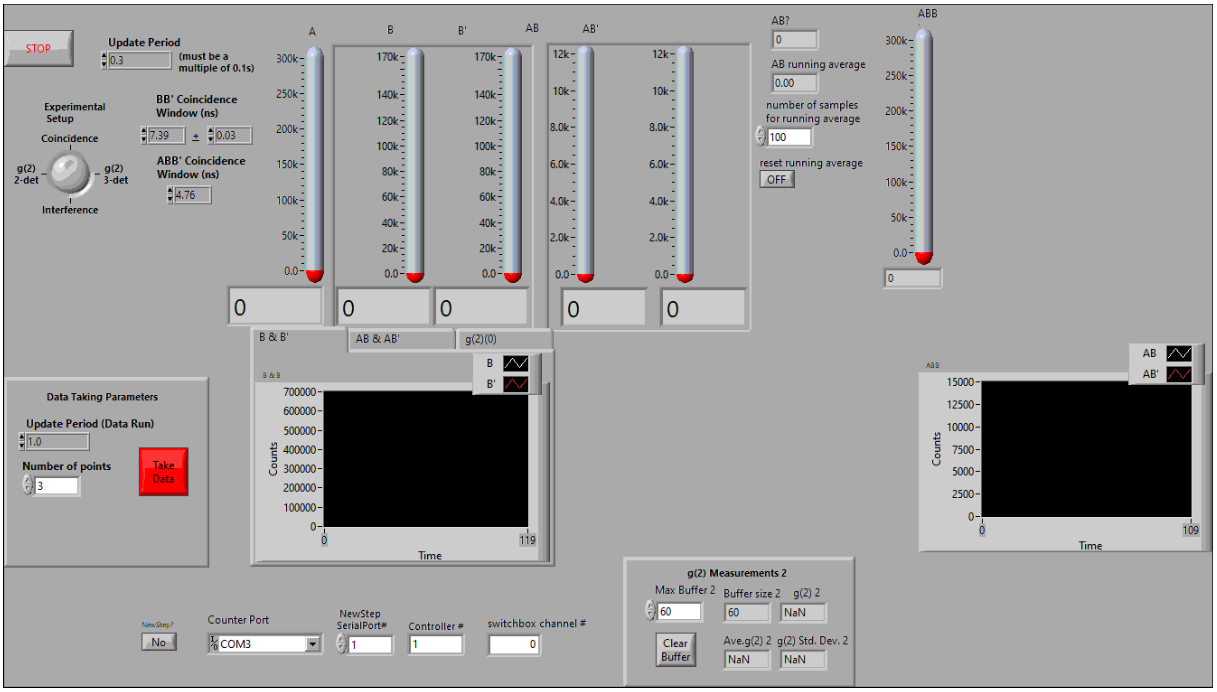

Figure 4: Front panel of the LabVIEW program used to measure the photon counts on the detectors. The functioning program was provided by [3].

As noted above, all the counts N here were averaged over 10 independent measurements. An ideal single-photon source gives $g_2(0) = 0$, while classical waves give $g_2(0) \geq 1$ [4].

Note that an attenuated laser source gives $g_2(0)= 1$. An attenuated laser mimics a classical wave by causing coincidences and failing the beam splitting test. This is due to the photons coming out of the laser being in a coherent state (more than one photon emitted simultaneously). So when the laser hits the beam splitter, some photons are transmitted and some are reflected like a classical wave. So, in this case, we cannot claim that we are considering one photon at a time. Heralded photons do so however, and so if $g_2(0) < 1$, then we have shown that our wave is not classical and that we have individual photons instead.

2.3 Creating and Confirming the Entangled State

We modified our previous setup (which had two collimators) as shown in Figure 7. We first added carefully calibrated linear polarizers and half-wave plates in front of the collimators (see Figure 8). Secondly, we swapped the BBO crystal for a new two sided thinner BBO crystal, which consisted of two layers of BBO crystals whose crystal axes are perpendicular to one another. The horizontally-oriented layer produces vertically polarized pairs of down-converted photons, while the vertically-oriented layer produces horizontally polarized pairs of down-converted photons. We also placed a quartz crystal on a rotating mount before the BBO crystal, and another half-wave plate behind the quartz crystal. The setup of these two crystals is shown in Figure 9. The quartz crystal and new BBO crystal were aligned using back reflection and fine-tuned to maximize detected photons. The image of the entire optical table we created is shown in Figure 10.

The linear polarizers near the collimators were set to horizontal transmission and the half-wave plates to 45 degrees so that vertically polarized photons were detected. The quartz crystal was oriented horizontally, and coincidences were maximized. Then, the half-wave plates were set to 0

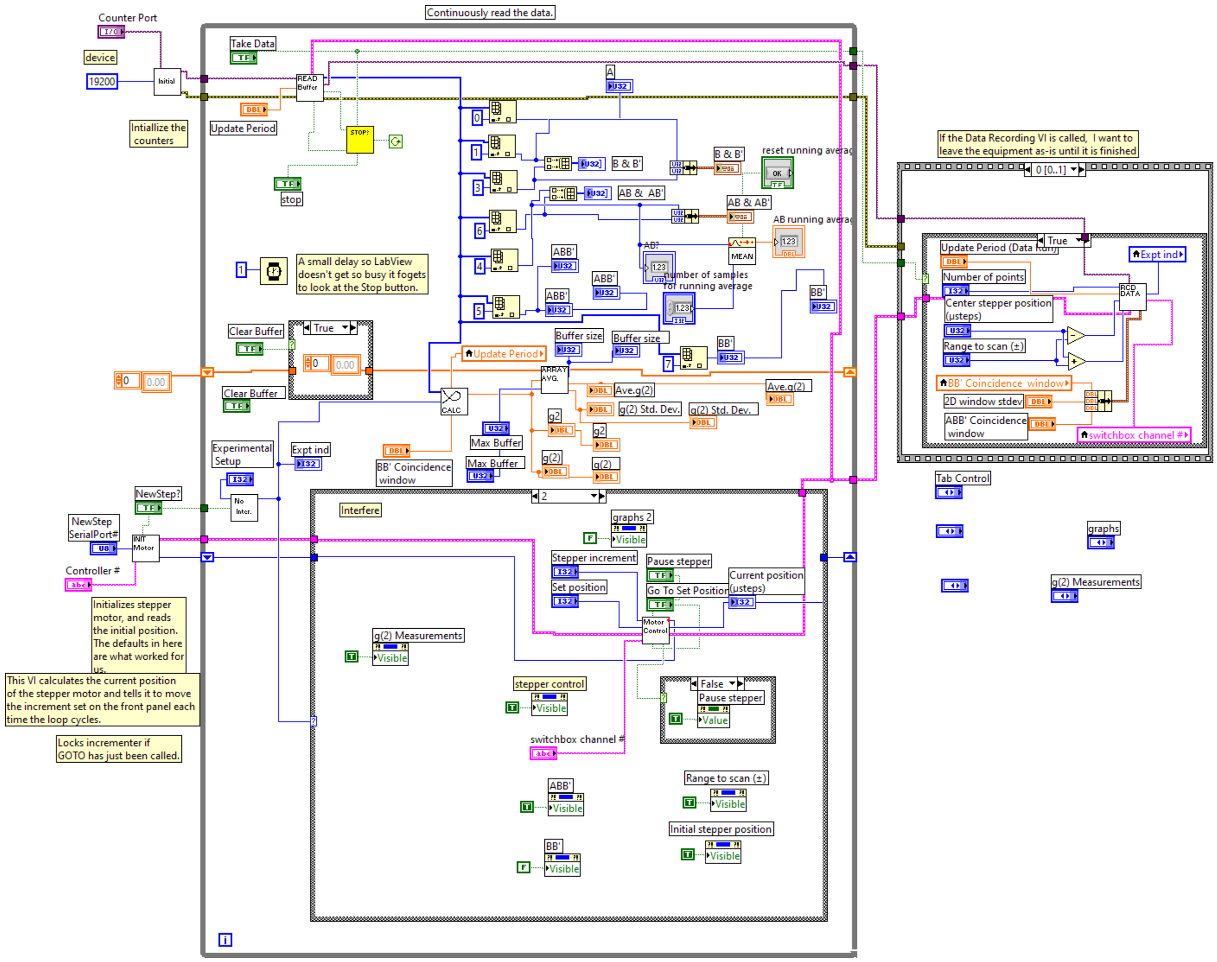

Figure 5: Block diagram of the LabVIEW program used to measure the photon counts on the detectors. The functioning program was provided by [3].

and zero coincidences were noted to confirm that we were no longer detecting vertically polarized photons.

The half-wave plate near the laser was then set to 45 degrees so the pump beam was vertically polarized. This time, the crystal was tilted to ensure that horizontally polarized photons were detected. The BBO crystal was tilted along the horizontal axis to maximize photon counts to confirm that horizontal coincidences were significant.

Now that we confirmed that the apparatus was working as intended and that we could detect horizontally and vertically polarized photons, we set up the entangled photons. The half-wave plates near the detector were set so that one detector detects diagonally polarized photons and the other detects anti-diagonally polarized photons. (22.5 degrees and 157.5 degrees.) The quartz crystal was carefully adjusted so that coincidences were minimized, as if the photons are entangled and one is diagonal, the other must not be detected at the anti-diagonal end- so we expect no coincidences. Accidental counts were subtracted off our results as usual.

Finally, we measured the CSHS parameter S to test Bell's inequality (Equation 2). To calculate S, we calculated the correlation parameter E for 4 different angles: $\theta_1 = 0°$, $\theta_1' = 45°$, $\theta_2 = 22.5°$ and $\theta_2' = 67.5°$. For each angle, the half-wave plate was oriented so that light of polarization θ was detected in a horizontal polarizer. The angles we used for the half-wave plate are given in Table 1.

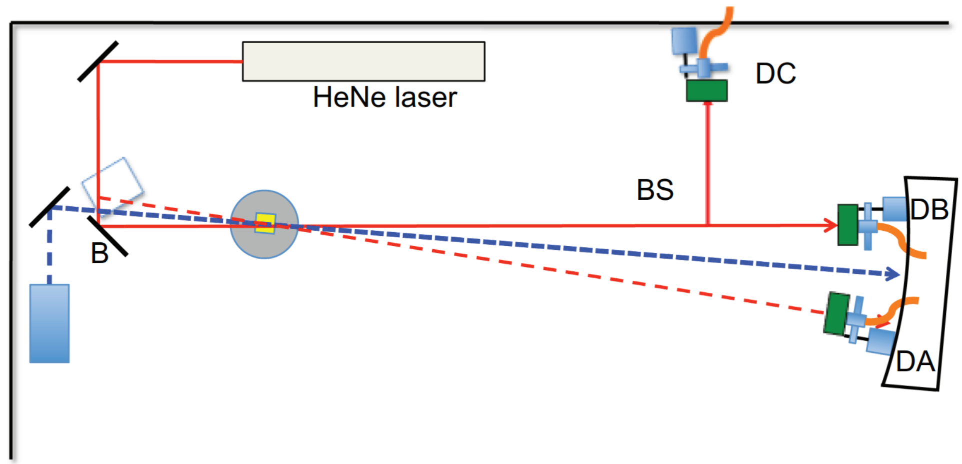

Figure 6: Setup for checking the existence of photons. A beam splitter and a third collimator were added to the initial setup shown in Figure 2.

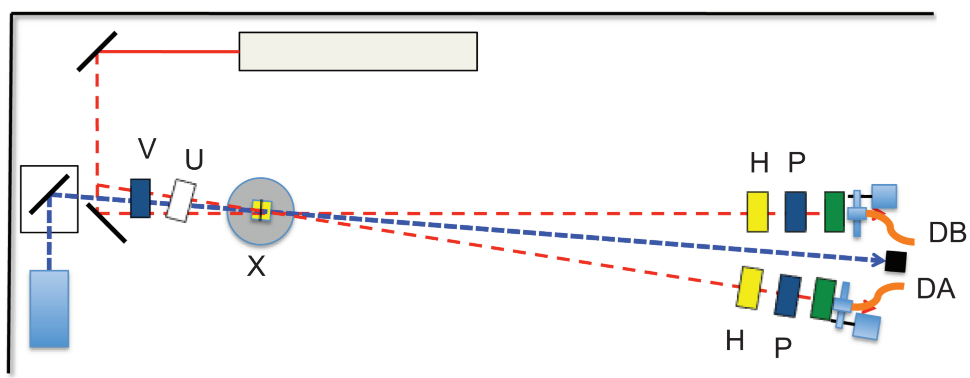

Figure 7: Setup for creating and confirming the entangled state of down-converted photons.

For each of these angles, the coincidences were noted and E was calculated. To account for the fact that the detectors do not have an ideal 100 % detection efficiency, we used the following *normalized* correlation parameter:

$$E(\theta_1, \theta_2) = \frac{N(\theta_1, \theta_2) + N\left(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}\right) - N\left(\theta_1, \theta_2 + \frac{\pi}{2}\right) - N\left(\theta_1 + \frac{\pi}{2}, \theta_2\right)}{N(\theta_1, \theta_2) + N\left(\theta_1 + \frac{\pi}{2}, \theta_2 + \frac{\pi}{2}\right) + N\left(\theta_1, \theta_2 + \frac{\pi}{2}\right) + N\left(\theta_1 + \frac{\pi}{2}, \theta_2\right)}, \tag{15}$$

where $N(\theta_1, \theta_2)$ is the number of coincidences with the two polarizer angles at θ_1 and θ_2 measured in a set amount of time. For our measurement, we used a time interval of 120 seconds for each of our E measurements. Then S was calculated using Equation (1), and its uncertainty using

$$\Delta S = \sqrt{\sum_{i=1}^{16} \left(\Delta N_i \frac{\partial S}{\partial N_i} \right)^2}, \tag{16}$$

where the uncertainties in the counts come from the statistical (Poissonian) error: $\Delta N_i = \sqrt{N_i}$.

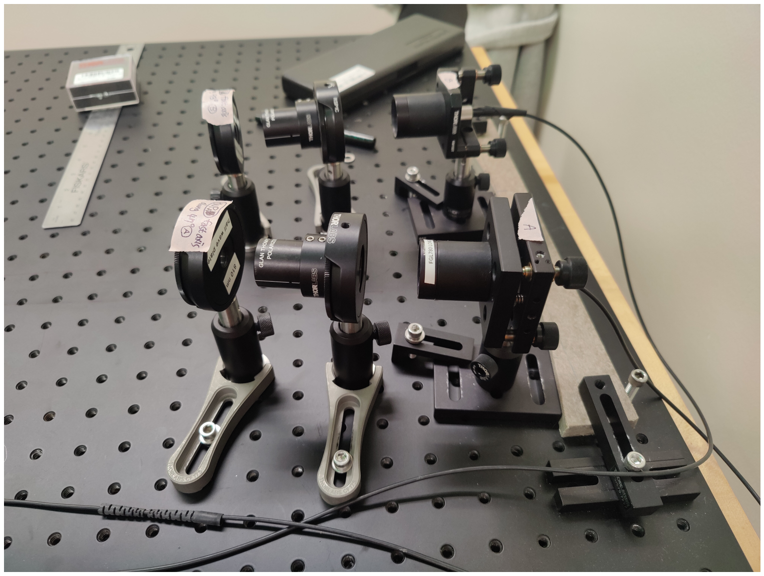

Figure 8: Setup of linear polarizers, half-wave plates, and collimators for the final setup.

θ	HWP angle	θ	HWP angle
$\theta_1 = 0°$	0°	$\theta_1 + \frac{\pi}{2}$	45°
$\theta_1' = 45°$	22.5°	$\theta_1' + \frac{\pi}{2}$	157.5°
$\theta_2 = 22.5°$	11.25°	$\theta_2 + \frac{\pi}{2}$	146.25°
$\theta_2' = 67.5°$	33.75°	$\theta_2' + \frac{\pi}{2}$	168.75°

Table 1: Table showing the polarization angles θ of the photons and their corresponding half-wave plate (HWP) angles needed to project the polarizations onto the horizontal polarization (this is because the polarizors (P) were set to only detect horizontal polarations). All angles are measured from the right-hand-side horizontal axis, in degrees, as we look in the same direction as the light propagation.

Each term $\Delta N_i \frac{\partial S}{\partial N_i}$ was calculated by:

$$\Delta N_i \frac{\partial S}{\partial N_i} = \sqrt{N_i} \frac{2(N_3 + N_4)}{(N_1 + N_2 + N_3 + N_4)^2} \quad \text{for } i\text{=1,2, and} \tag{17}$$

$$\Delta N_j \frac{\partial S}{\partial N_j} = \sqrt{N_j} \frac{-2(N_1 + N_2)}{(N_1 + N_2 + N_3 + N_4)^2} \quad \text{for } j\text{=3,4,} \tag{18}$$

where N_i is the i-th coincidence count of the 16 measurements. Other terms 5-16 are calculated the same way.

Since we were measuring the entangled state, we expect to violate Bell's inequality so S should be strictly above 2. In the next step, we oriented our quartz crystal vertically in order to break the entanglement and satisfy the CHSH inequality. Retaking the 16 measurements should give S strictly less than 2 as a nonentangled state doesn't violate the inequality. For each trial of the 16 measurements we set a 2-minute interval to collect data over.

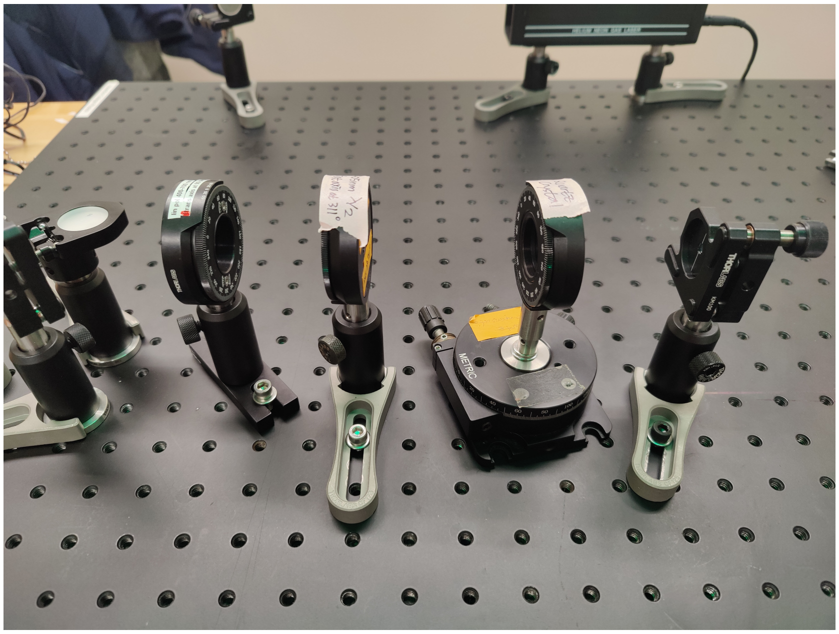

Figure 9: Alignment of the quartz crystal on a rotatory mount and the BBO crystal (right-most optic).

	Dark Counts	Laser On
N_A	5000	830000
N_B	4300	237000
N_{AB}	2	47000
N_{acc}	1	4600

Table 2: Measurements that demonstrated we have down-converted photons.

3 Results and Discussions

3.1 down-conversion of Photons

Our initial measurements of the counts at collimators A and B and coincidences (AB) are shown in Table 2. Dark counts, the measured counts with the pump beam turned off, are also reported. Accidental counts were calculated by Equation (12). The measured coincidences were more than 10 times larger than the accidental counts, thus we concluded that we successfully created and detected down-converted photons.

3.2 The Photon Exists

For our setup, we measured counts for collimators A, B, and C, and coincidences for AB, AC and ABC. Our counts were averaged over 10 trials of 2 minutes each, and our measured $g_2(0)$ and error were

$$g_2(0) = 0.65 \pm 0.10. \tag{19}$$

All quoted uncertainties in this report represent 95 % confidence interval, which is twice the error calculated. This showed that our $g_2(0)$ value was significantly below one, indicating that our laser

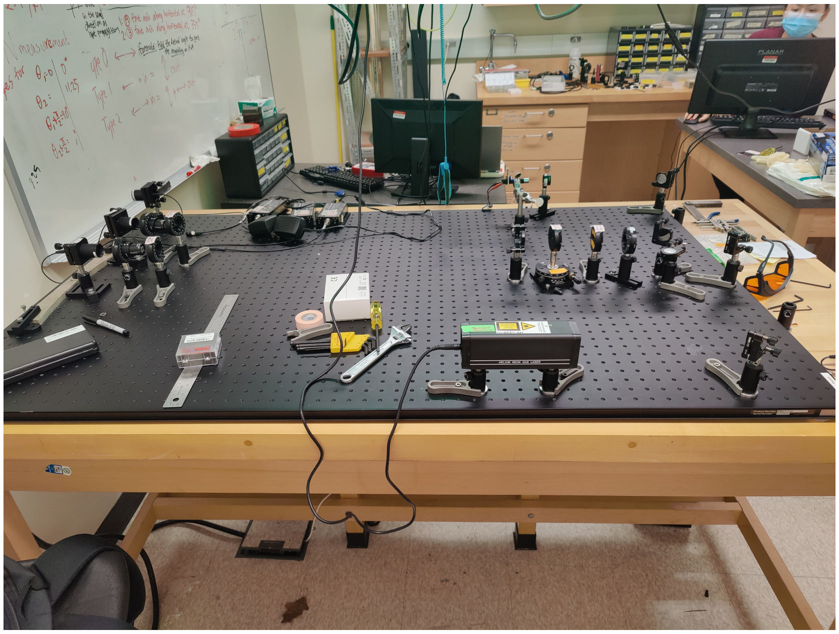

Figure 10: The entire optic table used to create and confirm entangled pairs of photons.

N_A	23207
N_B	19531
N_{AB}	6
N_{acc}	1

Table 3: Measurements for demonstrating that we have an entangled state taken during a 3-second interval.

was not a classical beam, and that we did indeed measure individual photons.

3.3 Quantum Entanglement and Bell's Test of Local Realism

We confirmed that we could detect vertically polarized photons when the linear polarizers were horizontal and the half-wave plates were vertical. We also confirmed that changing the half-wave plates to 0 degrees made it so that we could not detect any horizontal photons as expected.

After this, we set the first half-wave plate near the pump beam to vertically polarize the pump beam (the half-wave plate was set to 45 degrees), and oriented and adjusted the quartz crystal horizontally to ensure that we could also detect horizontally polarized photons in both detectors.

In the entangled state, one polarizer was set to 22.5 degrees and the other to 157.5 degrees. This allowed us to detect diagonally polarized photons on one detector and anti-diagonal polarized photons on the other. However, since our photons were entangled and thus they had the same polarization, we detected very low coincidences. Our measurements are shown in Table 3. The measurements were taken during a 3-second interval. The accidental count for this measurement was on the same order of magnitude of the coincidence counts we got. This confirmed that we indeed had photons in the entangled state (Equation 3), for which there is zero probability of jointly observing polarizations A and D, as we calculated in Equations (7).

In order to measure S, we gathered data from Table 1. For each trial $N(\theta_1, \theta_2)$ we counted for 2 minutes and noted the counts on collimators A and B. Also, we noted the coincidences AB and

i	N_{4i+1}	N_{4i+2}	N_{4i+3}	N_{4i+4}	E
0	1544	2589	79	64	0.9329
1	797	609	992	660	-0.0803
2	949	1299	727	663	0.2358
3	1565	1832	174	166	0.8181

Table 4: Coincidence counts (with accidental counts subtracted) and the four calculated values of E for the entangled state.

the accidental counts calculated using Equation (12). The final coincidences were used to calculate $E(\theta_1, \theta_2)$, which were used to calculate S and its error using Equations (1) and (16). Our count measurements are shown in Table 4 and our measured S was

$$S = 2.07 \pm 0.05. \tag{20}$$

The maximum accepted value to satisfy local realism was outside the 95% confidence interval, and thus we confirmed that our entangled photons violate the theory of local realism.

In the second state, which was *unentangled*, the same 16 measurements were repeated and S was recalculated. Our measurements are shown in Table 5, and our measured S for the unentangled state was

$$S = 1.25 \pm 0.04, \tag{21}$$

which was clearly smaller than 2 and thus satisfied the CHSH inequality, given by Equation (2). This confirmed that by rotating the quartz crystal by 90 degrees, we broke the entanglement and rather had a mixed state.

Our coincidences could have been higher because one of our collimators was significantly more sensitive than the other, which made it so that it detected more photons than the other, and this means we were possibly missing out on some coincidences that we would have measured if our other collimator was also as sensitive.

Additionally, orienting the pump laser exactly straight was very challenging as it had an unstable mount that would easily move. It was adjusted by hand to be approximately straight, but using tighter and more precise equipment would lead to a straighter and better pump beam.

The process of back projection was often difficult to do precisely because our class 3R laser was extremely dim and wide, which made it hard to pass it's center through the crystal. Using a more precise laser for back projection would allow us to orient our collimators better and possibly measure more coincidences.

We also noticed that light reflected off our t-shirts and light from the computer screen affected our measurements quite drastically. To minimize this, we turned the brightness on the screens down, tilted them away from the apparatus and wore darker colored clothes that absorb more light.

4 Conclusion

We performed a series of experiment to create entangled pairs of photons that violate the CHSH inequality. The maximum allowed value of S that satisfies local realism was lower than our measured value of S and was outside the 95% C.I. of our measured S. This demonstrates that entangled photons measured at certain polarizations can indeed violate local realism, thus confirming once more that the theory of local realism does not hold. This important result indicates that observing

i	N_{4i+1}	N_{4i+2}	N_{4i+3}	N_{4i+4}	E
0	1562	4141	113	192	0.8986
1	618	1554	1083	2847	-0.2882
2	764	3764	3033	661	0.1013
3	2091	1657	1555	2489	-0.0380

Table 5: Coincidence counts (with accidental counts subtracted) and the four calculated values of E for the *unentangled* state.

the state of one of the entangled photons immediately determines the state of the other photon without having to perform extra measurements, and thus proves that quantum mechanics is non-local.

In future experiments, more careful experimental considerations can be taken in order to measure a higher CHSH parameter, and thus more significantly violate the CHSH inequality. In our experiments, we had some background light coming from the head lights in the hallway and the computer monitors. We saw that our measurements fluctuated quite a lot in some cases. Covering all those unwanted light sources with black screens or cloths would help obtain much more stable measurements. Moreover, we had 1° to 2° errors in the fast and transmission axes of our half-wave plates and linear polarizers, but one could calibrate them more accurately. The angles of the quartz and the BBO crystals can also be calibrated the same way. We used the axes given by the manufacturer. With these extra considerations, much stronger violations of the CHSH inequality could be achieved.

Acknowledgement

We sincerely thank Professor Walter F. Smith for providing helpful discussions, guiding us through this long experiment, and most importantly for this wonderful lab course!

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A Adi and Charlotte's Tips for Future Groups

A.1 Calibrating Optics

This experiment is extremely sensitive to the polarizations, and thus it is very important to make sure that all the optics that affect the polarization are well calibrated. Before you start your experiments, check the fast axes of all the HWPs and the polarization angles of all linear polarizers!

A.2 Angles for Half-Wave Plates

Be careful when you try to find some generalized formula for finding an angle for the HWPs–it's more complicated than you think, and the formula you come up for the first few angles will *not* work for other angles. We suggest that you draw pictures for each angle. Recall that the polarization of the light is reflected over the fast axis. Also, make sure that you use a consistent system for measuring angle. We measured our angles from the right-hand sind of horizontal axis when we were looking down the laser path (i.e. looking in the same direction as the light propagation direction). And finally, don't forget that the *markings* on the optics are generally different from the actual angles you want to set the fast axes to!

For example, in order to detect a photon with polarization at 45° above horizontal, the fast axis of the HWP should be set to 22.5° above horizontal. For us, that meant that we had to set the marking of our HWP to 339.5°.

A.3 Laser Setup and troubleshooting

Firstly, you want to safely turn on and off the laser using a power supply. To do so, you can connect the negative terminal of the power supply to ground and then to the laser itself. To turn the laser off then, simply connect the ground to the positive terminal which will reduce the input voltage to 0V, turning off the laser safely.

The laser comes with a small square holder that is placed on top of the activation button. If your laser has no visible output, it is possible that the holder has slipped off the button and has turned off.

You will probably have to reflect the laser beam off a few mirrors during your setup. This will create an elliptically polarized beam that behaves strangely with the equipment. To avoid this issue, always place your polarizer after all the mirrors so that the resulting beam is linearly polarized.

There are two main ways to make sure your laser is aligned with a particular collimator. The first, and more accurate way to align the laser is using back-projection. Use a Class 3R red laser with a fibre optic cable passed into the collimator. Make sure the output beam of the Class 3R laser aligns perfectly at the mouth of the laser. To do so, you might find it useful to cut a small hole in a piece of paper through which you can pass the laser to align it. The second method is called back reflection, by which you simply orient the collimator in such a way that the reflection of the laser in the collimator passes back into the beam. Always make sure you use both back projection and reflection after you place a collimator or adjust a collimator in any way.

Use a lens to collimate the laser if the diameter is too large and it is clipping on your crystal. You can make sure it is clipping by looking for the shape of the laser on the wall. If the laser is rectangular, it means that it is clipping on the sides of the rectangular crystal.

Finally, and most importantly, NEVER turn on the lights while a collimator is active or plugged in. Always shut down labVIEW and unplug the collimators before turning on the lights. While using the collimators, use the provided green LEDs in the lab.

A.4 Changing to the new BBO crystal

After you swap the BBO crystal to the new one, you basically have to readjust the collimators to maximize the number of counts you get. Recall that you had to change the horizontal adjustment of the BBO crystal to maximize the count on one collimator first. Then, you change the height and the position of the other collimator along the curve in order to also maximize that height. When you do this, don't forget to take out the linear polarizers and the HWPs in front of the collimators because some combination of angles of those components might significantly decrease the counts, which is not what you want.

After you think you've maximized the counts on both collimators, try adjusting the horizontal adjustment of the BBO crystal slightly to make sure that *both* collimators hit their maximum counts at the same time.

A.5 LABVIEW troubleshooting

You might notice that one of your readings for collimator A or B is significantly higher than another one. Collimators cannot always be fixed, but you can try a few things to determine the root cause of the issue.

The problem can either lie with the collimator, your laser alignment or the fibre optic cable. First, always test your alignment, as this is the easiest fix. If your detectors have significant discrepancy despite your best attempts at alignment, you can try to swap the fibre optic cables of the collimators. You can also try swapping the collimators themselves. It is possible that one collimator is simply less sensitive than the others, and you will have to run your experiment with it. This is OK, and you can certainly achieve entanglement with two collimators with very different sensitivities.

You will need your labVIEW program to measure photon counts for collimators A, B and C, and also measure coincidences for AB, AC and ABC. These are all included in the coincidence software, however they might not be visible on the screen. To make them visible, you can visit the block diagram and locate the graph/thermometer_chart labelled AC, or ABC... You can make this item visible by connecting it to the wire coming out of the AC or ABC port. If things are wired correctly, your plot will show up in labVIEW.

Always make sure to use the correct COM port(COM1) when taking measurements, and also to only use time intervals that are a multiple of 0.1s.

It is important to not that the counts shown are not averages. The collimators collect the number of photons in a certain time span and display it to you, so if you need an average measurement you need to take data manually and divide by the number of trials.