

The Density of Modes- Background



Figure 1: A March 2023 Landslide in San Clemente, California [1]

Landslides cause enormous devastation but are poorly understood compared to other natural disasters.

We are attempting to verify the underlying physics behind the density of modes in granular matter and hope that this will allow us to predict failure events in geophysical systems.

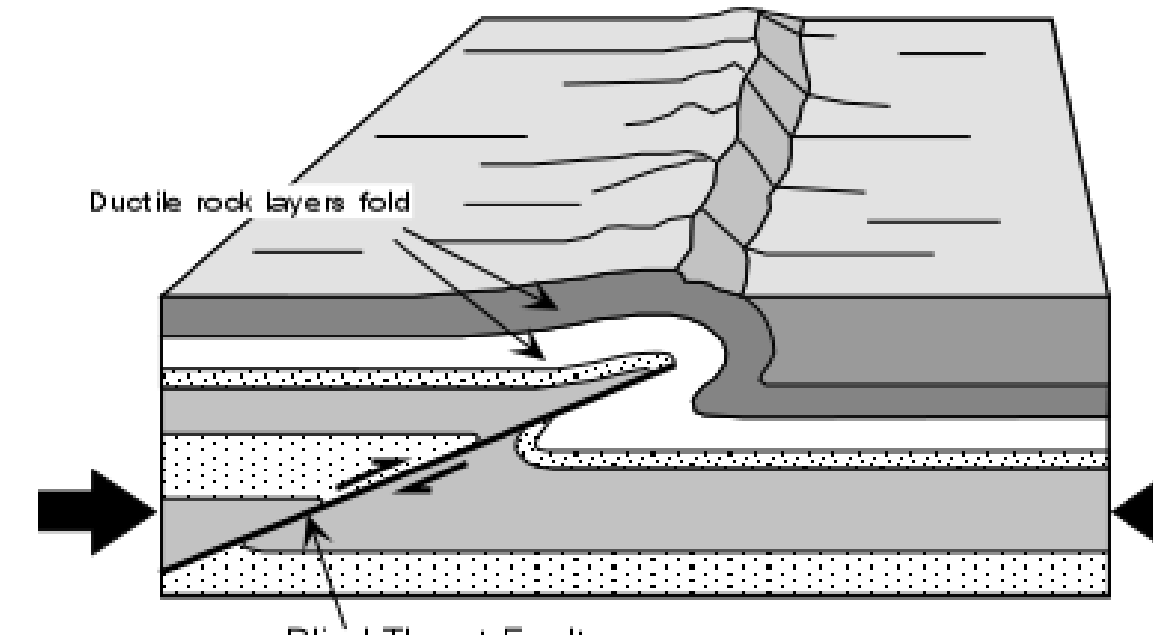


Figure 2: A diagram of a seismic fault [5]

What is the Density of Modes?

The density of modes ($D(\omega)$) is a quantity that describes the number of modes per unit frequency (ω). In other words, it tells us how many possible ways the system can respond at a given frequency.

In Debye Scaling (the black line on Fig 3)- we expect $D(\omega) \propto \omega^{d-1}$.

In a jammed granular system (Fig 3b) we observe an excess of low frequency modes relative to Debye scaling until some characteristic frequency ω^* . [6] This excess of low frequency (floppy) modes can clearly be seen in Fig 4.

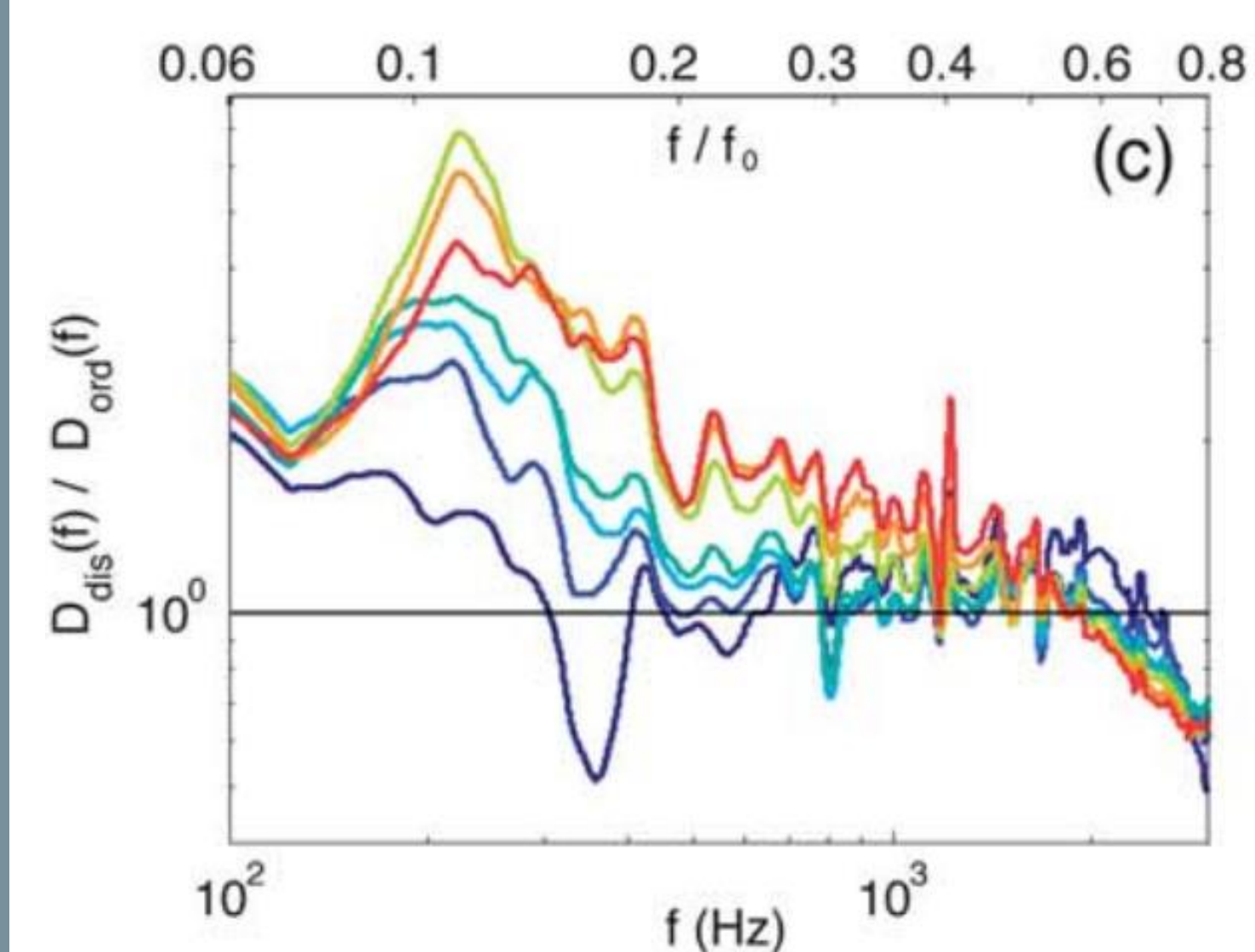


Figure 4: Dividing Fig 3.a. by 3.b. [2]

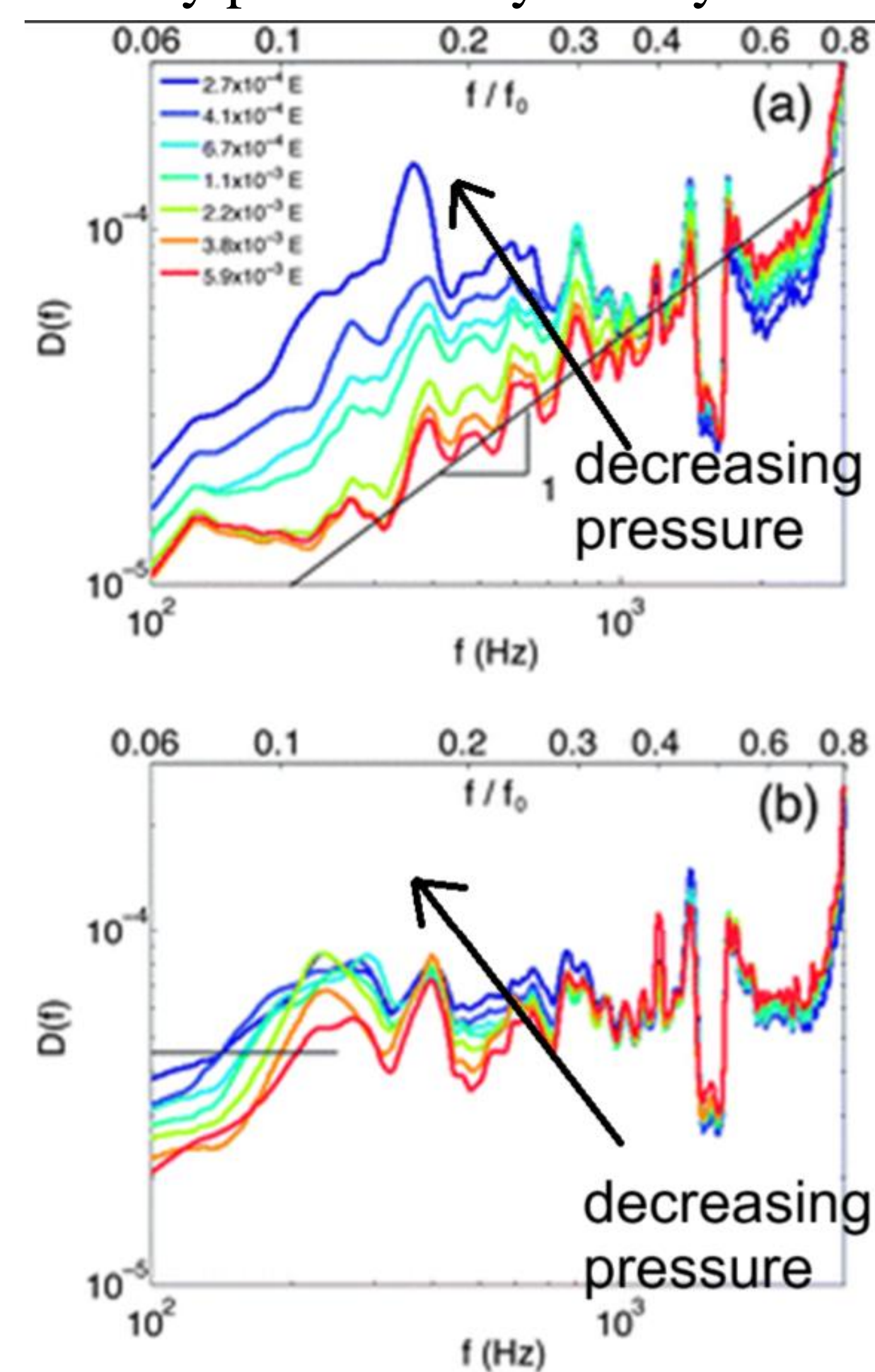


Figure 3.a. and 3.b.: The DOM at 7 pressures for a) an ordered system b) a disordered system (Debye scaling is the black line) [2]

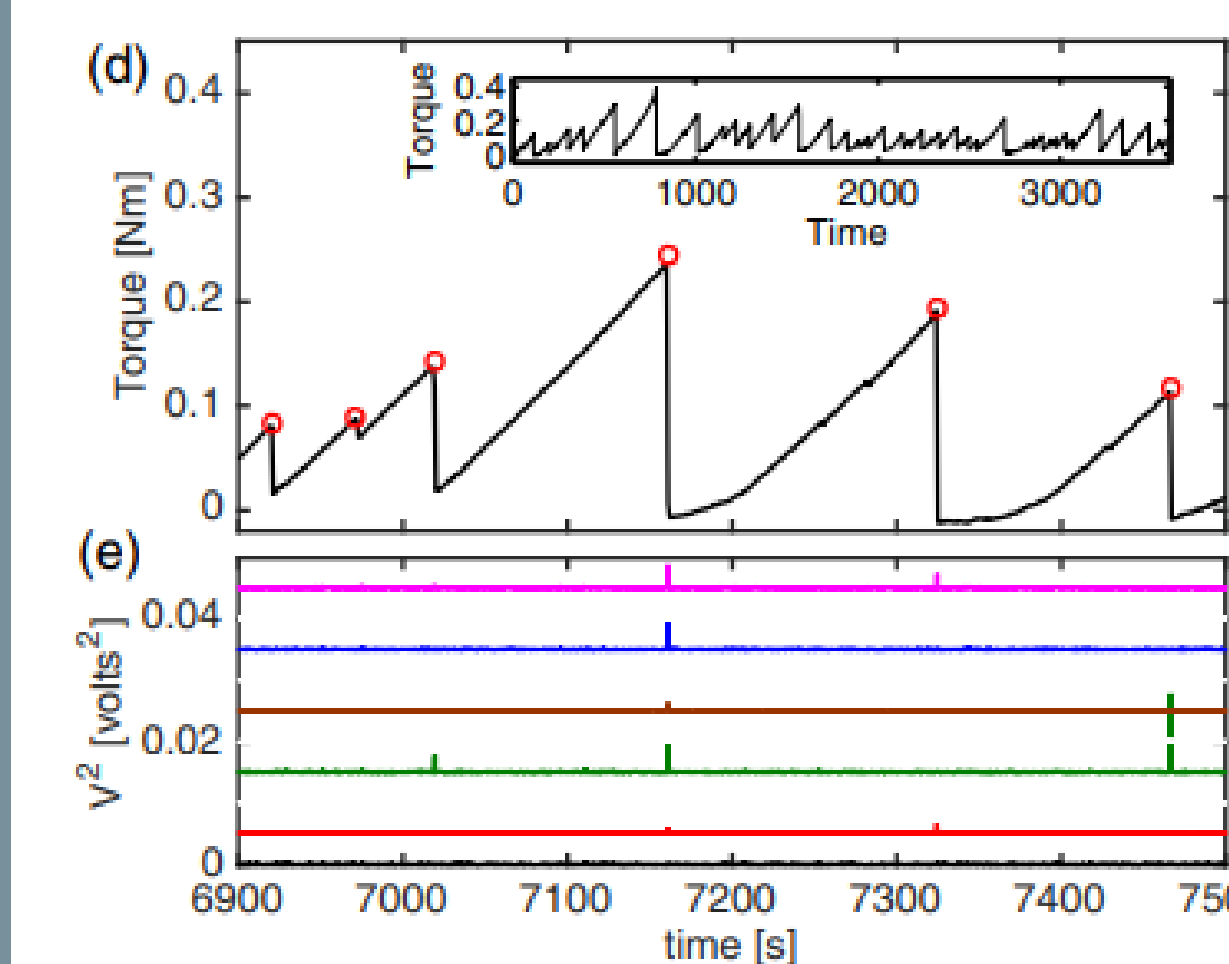


Figure 5: Showing how piezoelectric sensors can detect when a granular material fails (marked in red) [4]

How does the DoM tell us about sediment failure?

We are interested in how the value of ω^* changes as the system is disturbed. Figure 5 shows a system that was stressed into slip-stick failure.

As the system approaches failure, the pressure rises and so does ω^* , allowing us to detect when it fails just using the density of modes. [4]

Our goal is to use ω^* from the density of modes to **tell us about the state of the granular system.**

The Density of Modes- Analysis

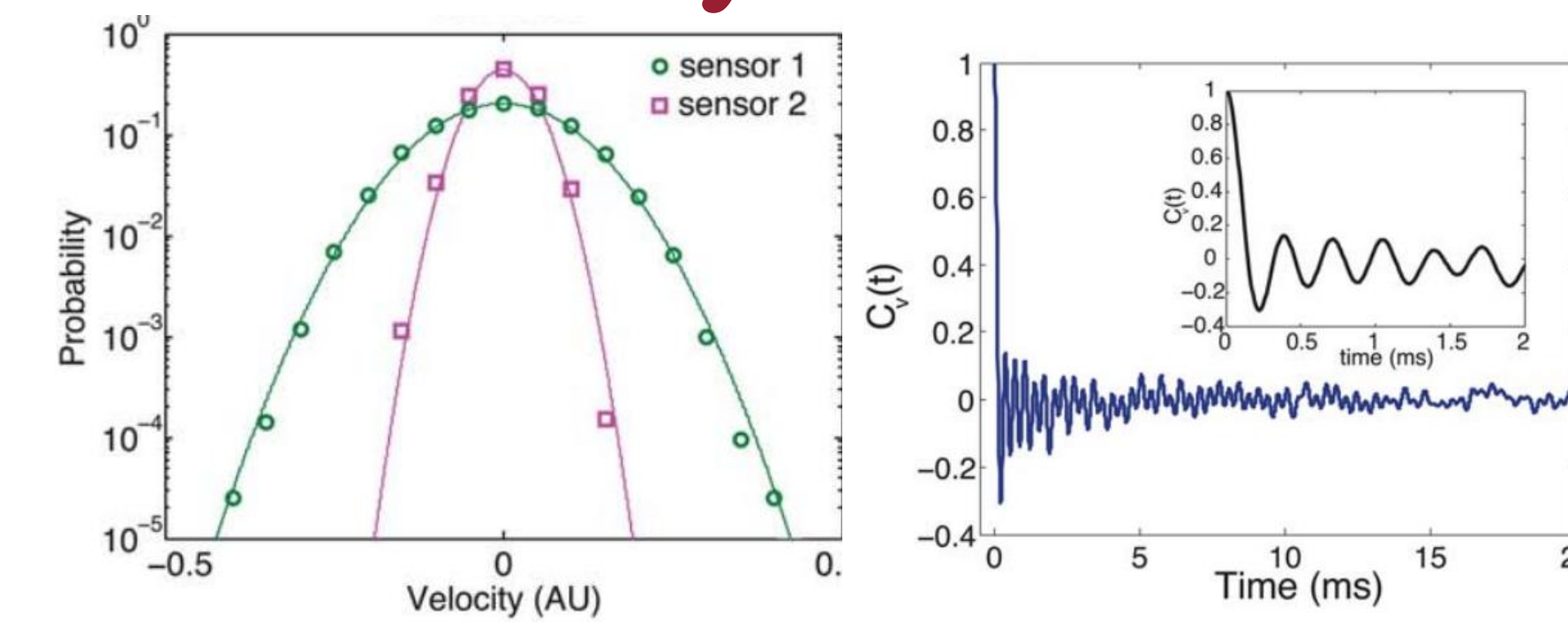


Figure 6: a) An example velocity distribution; b) a velocity auto-correlation function. [2]

To measure the DoM we will:

1. Collect the velocities of the particles. (Eqn. 1)
2. Put together a velocity auto-correlation function.
3. Take an FFT using this function.
4. Take the real part of this FFT, which is the DoM. (Eqn. 2).

Once we measure the DoM, we want to use it to **understand the state of the material.**

$$C_v(t) \equiv \frac{\sum_i \langle v_i(\tau+t) \cdot v_i(\tau) \rangle_\tau}{\sum_i \langle v_i(\tau) \cdot v_i(\tau) \rangle_\tau}$$

Equation 1: The Velocity Autocorrelation function. [3, 4]

$$D(f) \equiv \int_0^\infty C_v(t) \cos(2\pi ft) dt.$$

Equation 2: The Density of States [3, 4]

Mini DoMinator Design

The Mini DoMinator is a modular sample-collecting, storage, and testing system designed to measure the density of modes of small granular samples.

First, a sample of granular matter is collected in a small aluminium chamber. We cap the chamber with different types of caps that allow us to modify or acoustically listen to the sample.

The compression cap: Is used to compress the sample inside its chamber. (Figure 8)

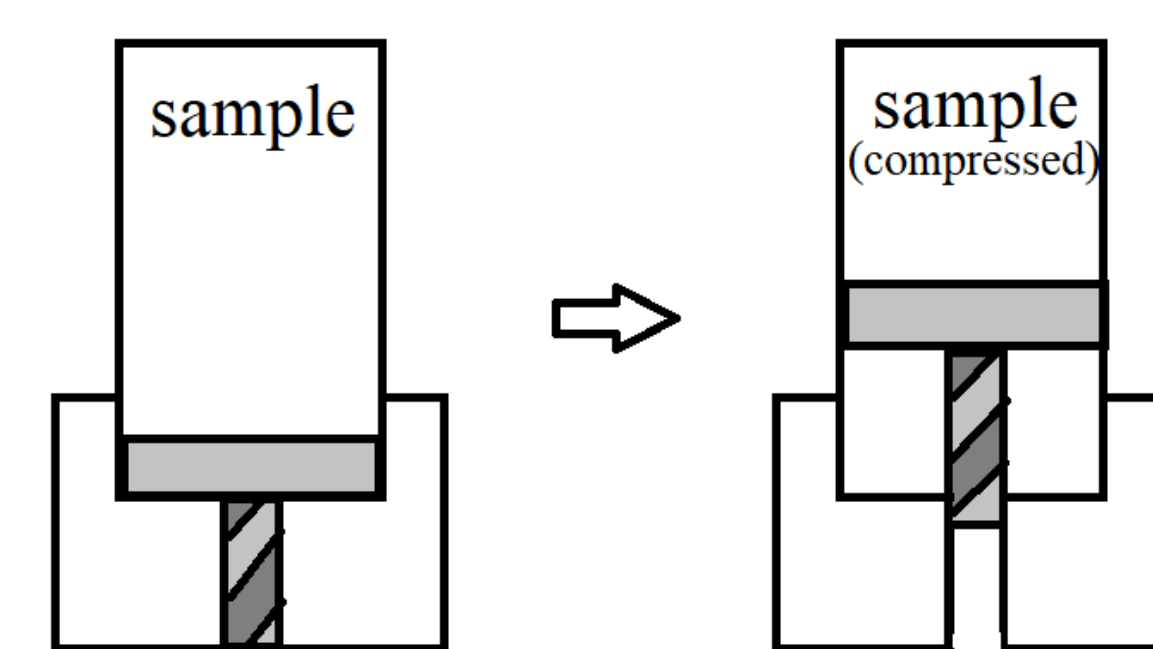


Figure 8: Using the compression cap.

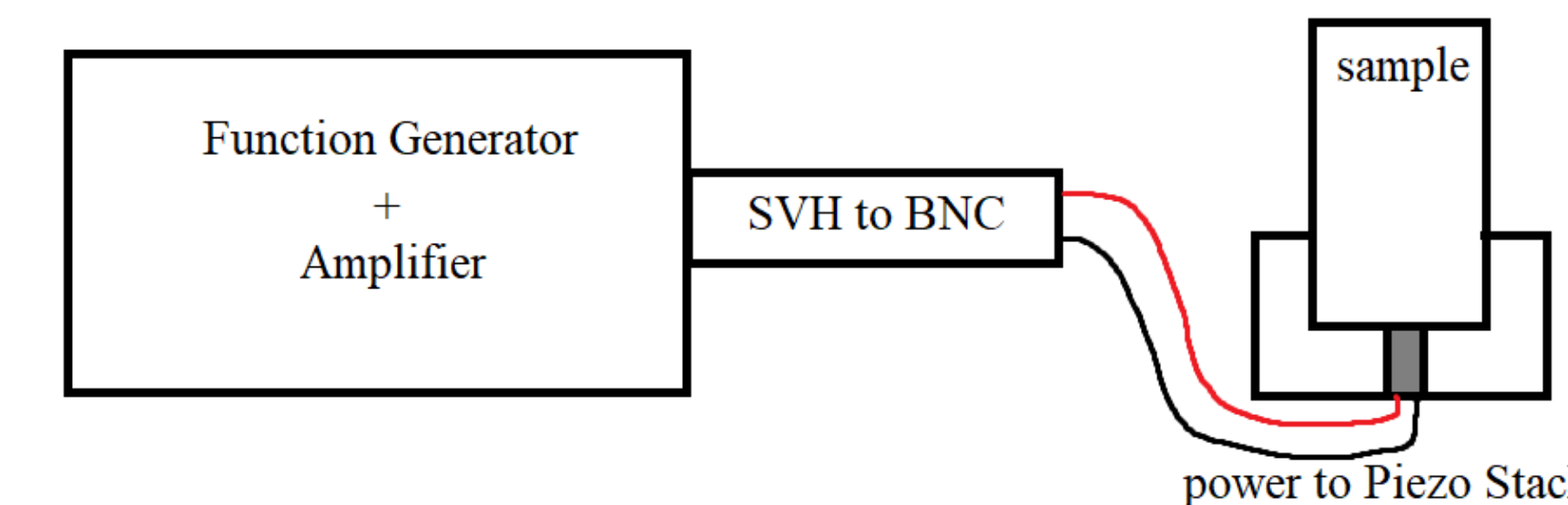


Figure 7: Diagram showing the interface between the function generator and driver cap.

The driver cap: Is fitted with a piezo-stack that can drive the sample at a certain frequency. We can also drive the sample using a shaker if needed. (Figure 7)

The reader cap: contains a piezoceramic crystal that generates charges proportional to the forces exerted on it by the sample in response to acoustic driving. (Figure 9)

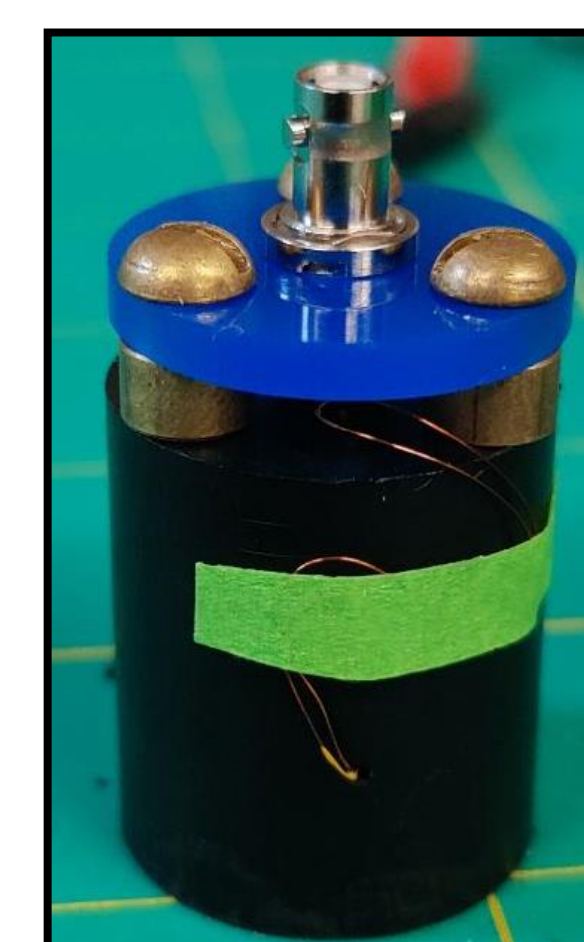


Figure 9: The Reader Piezo Cap.

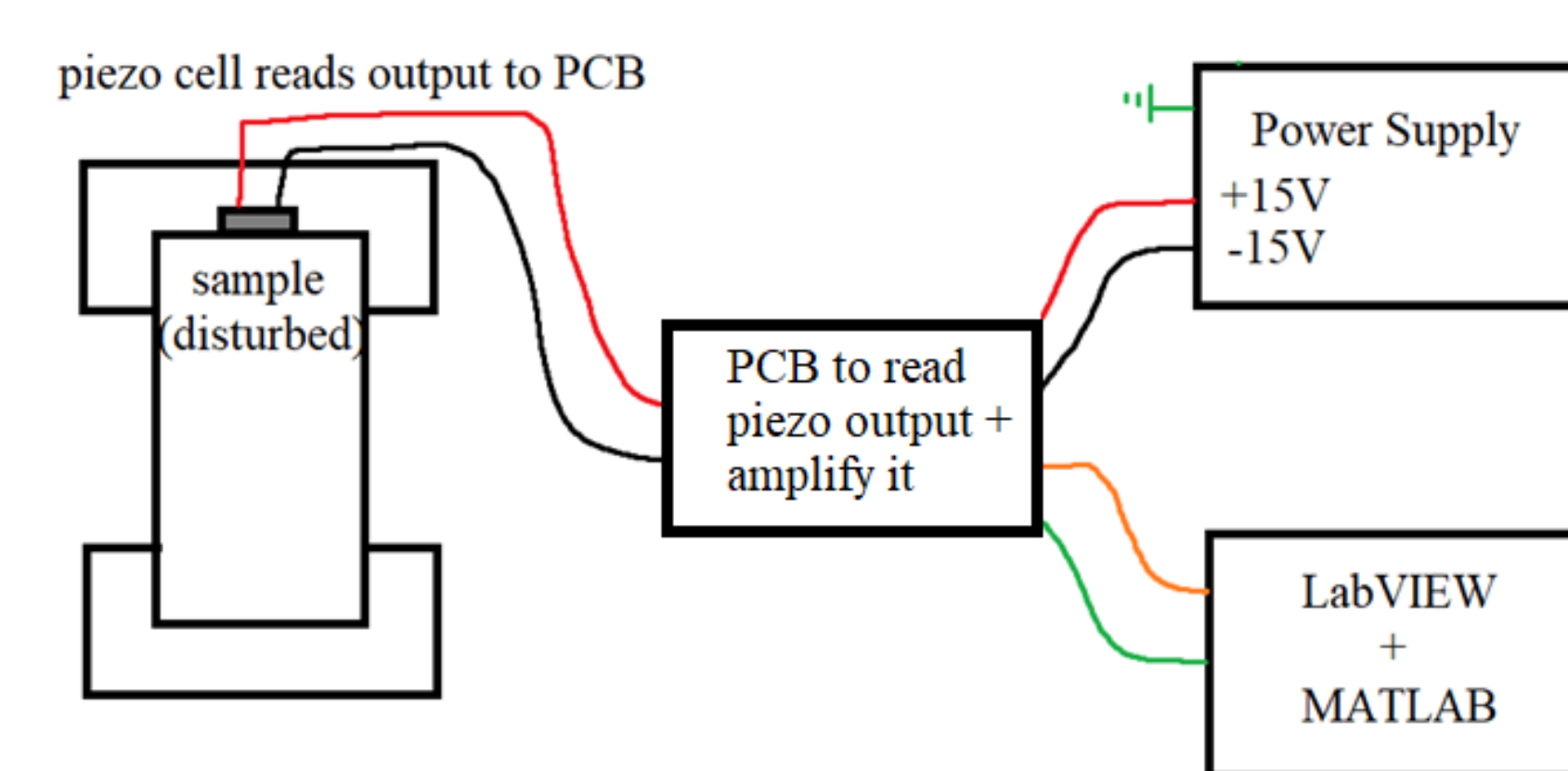


Figure 10: Diagram of the circuit working with the reader cap.

Finally, our circuit is designed to work with the reader cap to convert the piezoelectric charge to a voltage which we measure.

Field Data

The group visited Schenk Forest near North Carolina State University to collect samples and test equipment.



Figure 11: Beach at Scripps Institute of Oceanography, where initial geophone measurements were carried out



Figure 12: Professor Brzinski taking geophone measurements at NCSU

We collected data from a variety of environments. Figures 11 and 12 show our data collection sites at UCSD and NCSU.

Some of our data was collected using geophones while we manually disturbed the soil. The geophones measure the velocity of the particles directly rather than measuring the voltage (which is then to be converted to velocity).

Outlook + Future Plans



Figure 13: The team taking field data

Project Goals:

The DoM team is working on using the Density of Modes to assess the state of geophysical systems and forecast their failure events.

To do this, we collect field data and measure its acoustic response using piezoelectric sensors or geophones.

We then calculate the density of modes and perform tomography to better understand the state of the material.

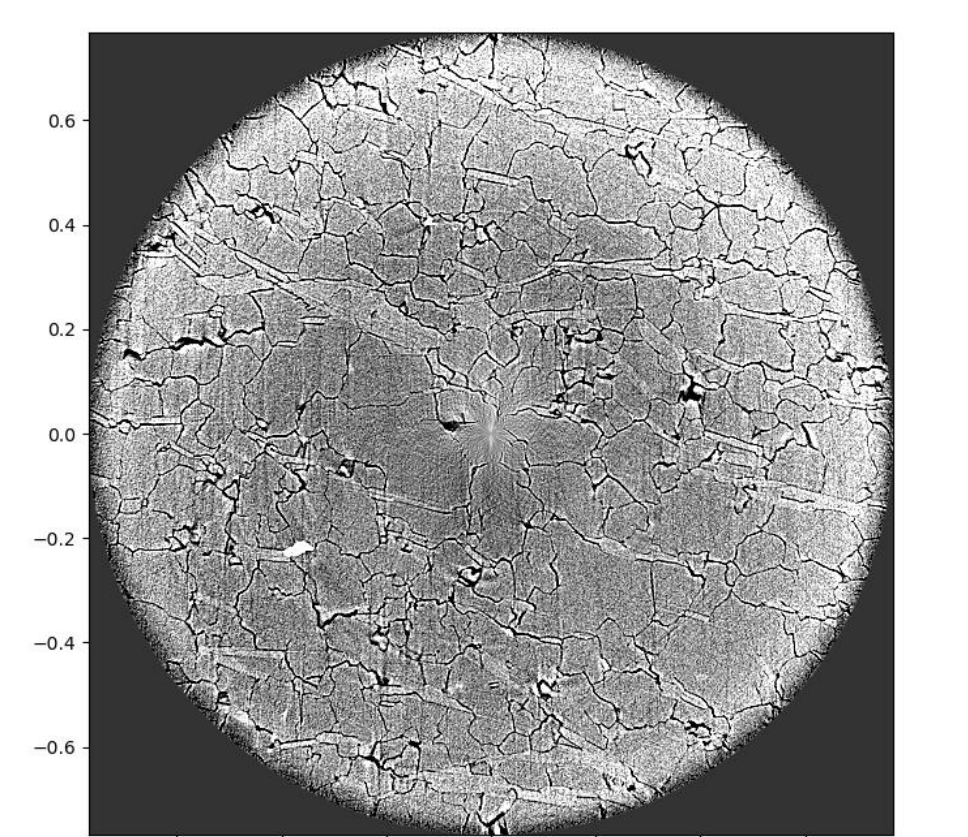


Figure 14: x-ray image of sandstone sample

References

- [1] [5] "Landslide Leaves Evacuated Residents' Futures in Limbo." San Clemente Times, 22 Mar. 2023.
- [2] Owens and Daniels, 2013, "Acoustic measurement of a granular density of modes", Soft Matter, 9, 1214-1219.
- [3] Dickey, J. M., and Arthur Paskin. "Computer Simulation of the Lattice Dynamics of Solids." Physical Review Journals Archive, American Physical Society, 15 Dec. 1969.
- [4] Theodore A. Brzinski, III, and Karen E. Daniels. "Sounds of Failure: Passive Acoustic Measurements of Excited Vibrational Modes." Physical Review Letters, American Physical Society, 25 May 2018.
- [5] Nelson, Stephen. "Natural Disasters." EQ Case Histories.
- [6] Xu, Ning, et al. "Excess Vibrational Modes and the Boson Peak in Model Glasses." Physical Review Letters, vol. 98, no. 17, 24 Apr. 2007.